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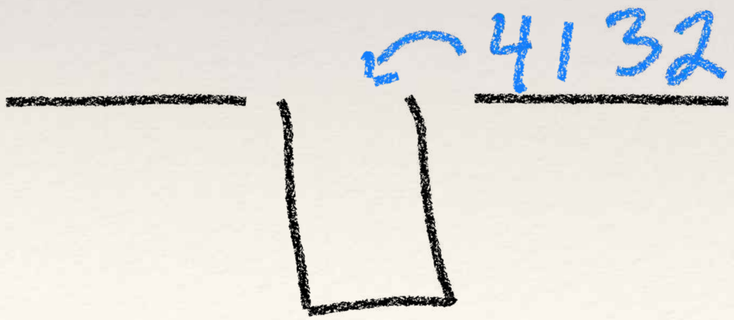

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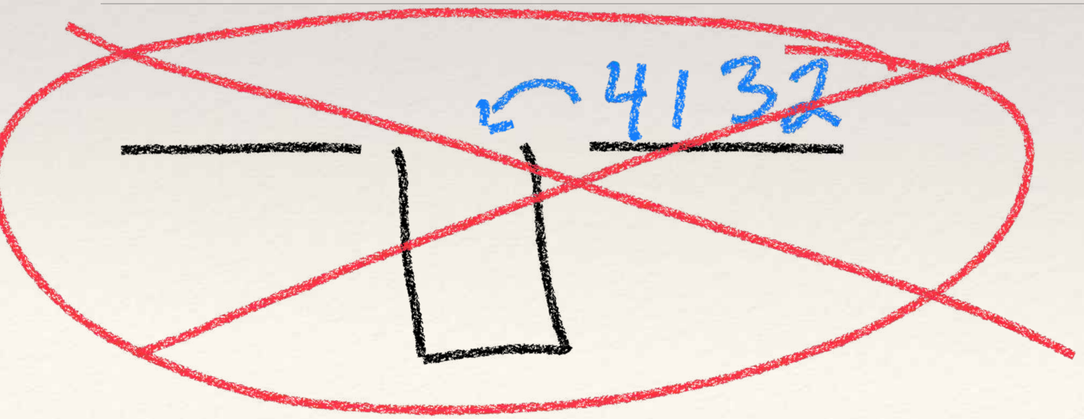

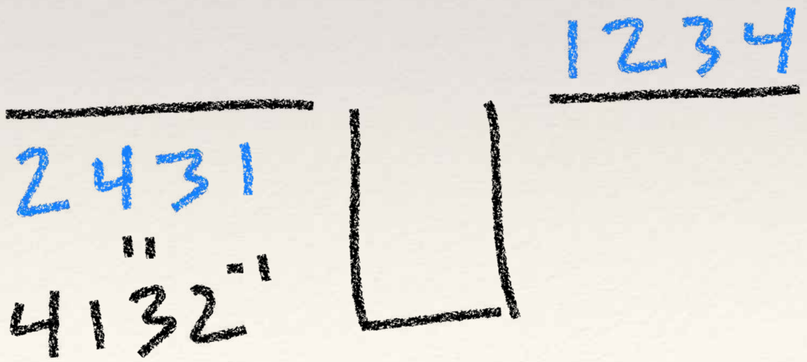

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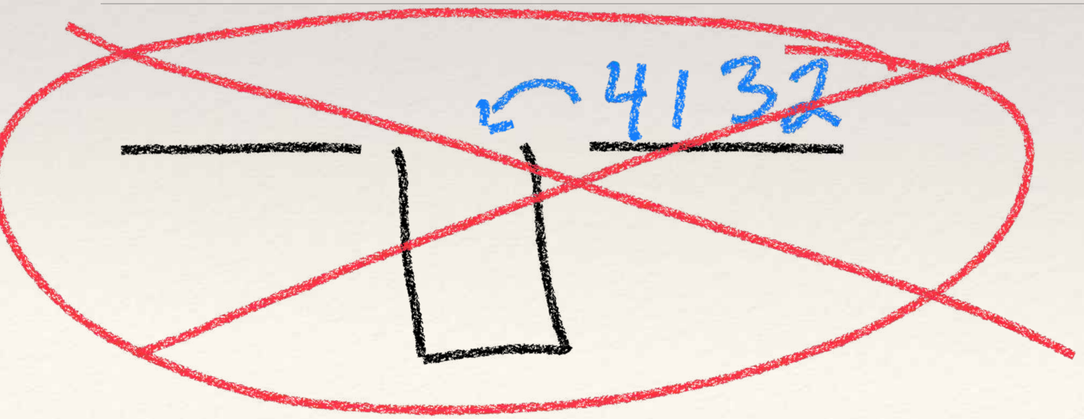

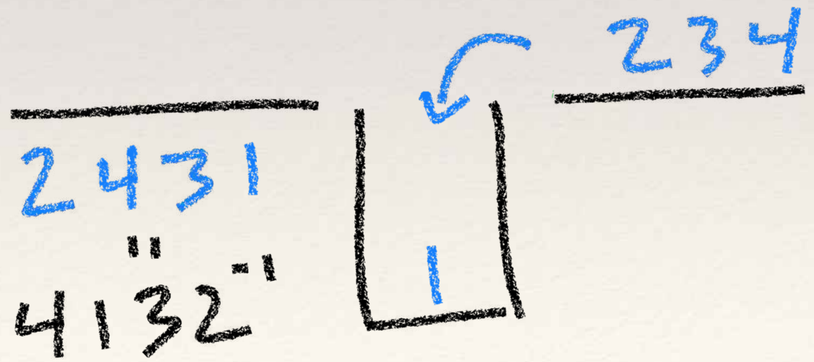

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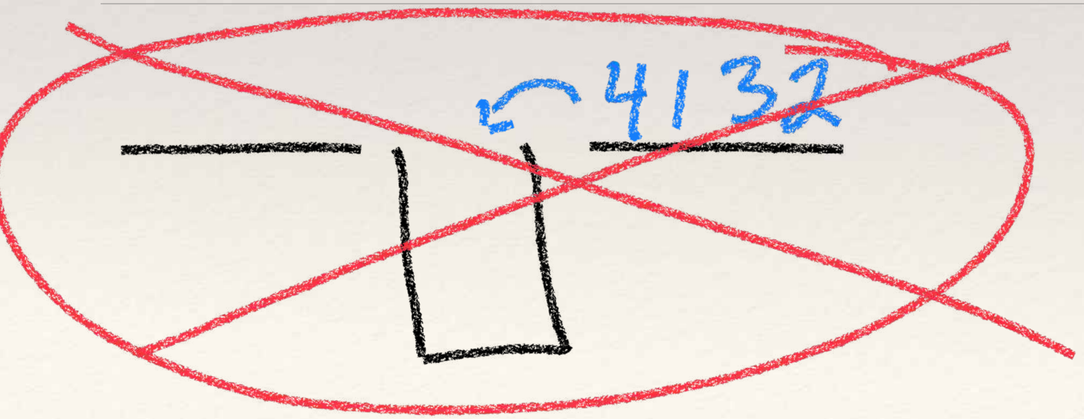

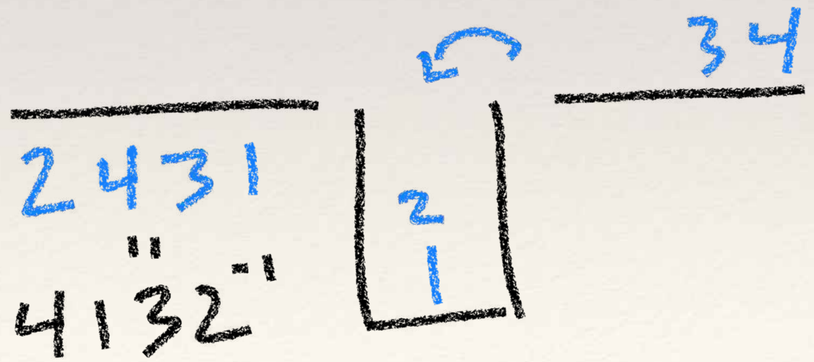

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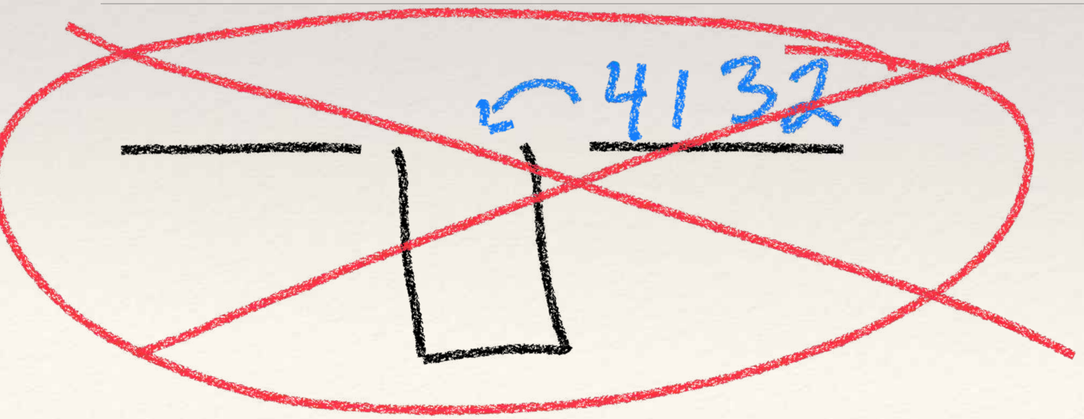

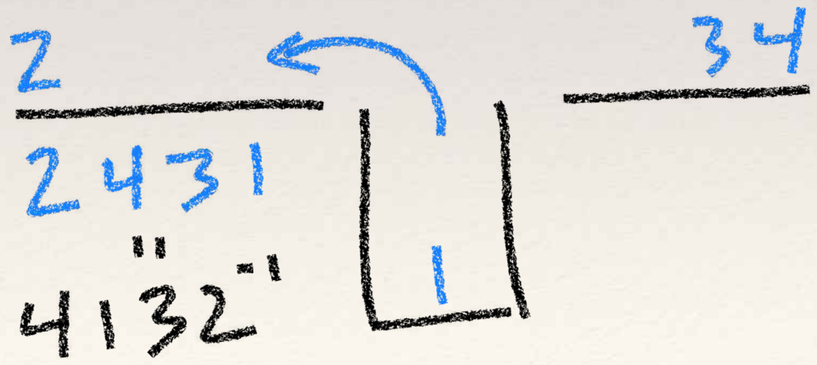

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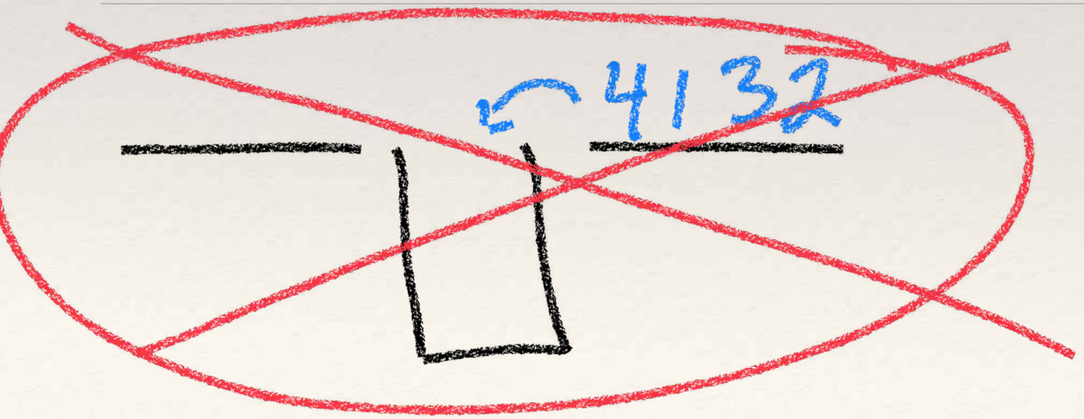

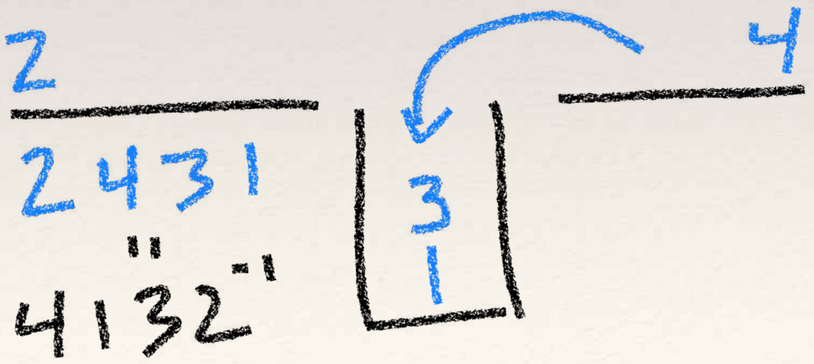

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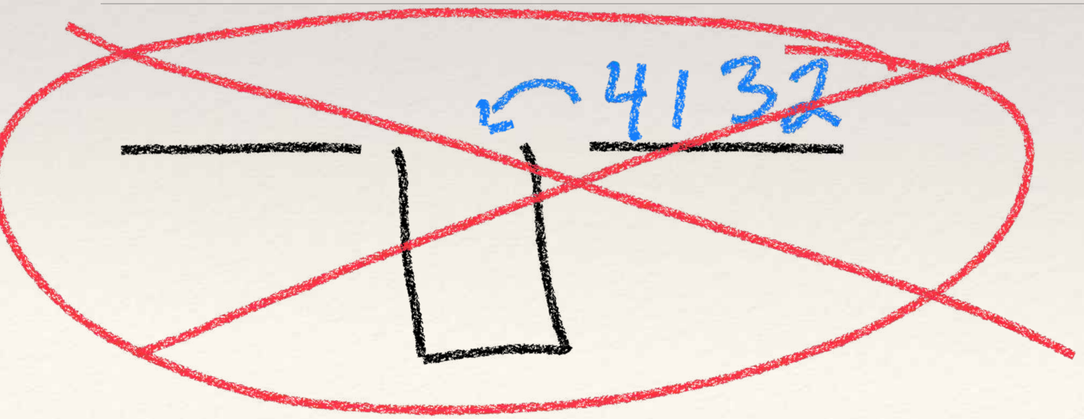

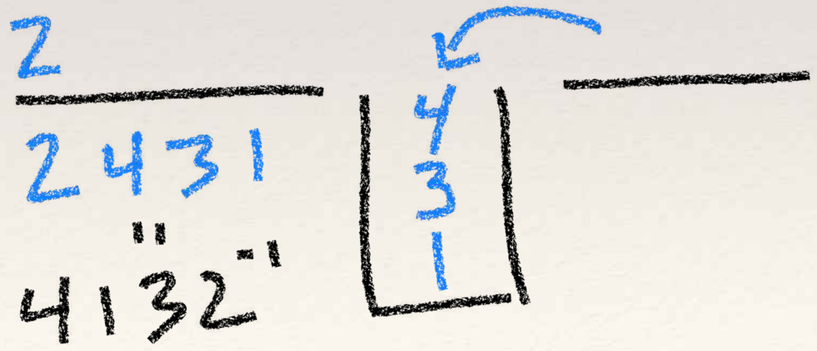

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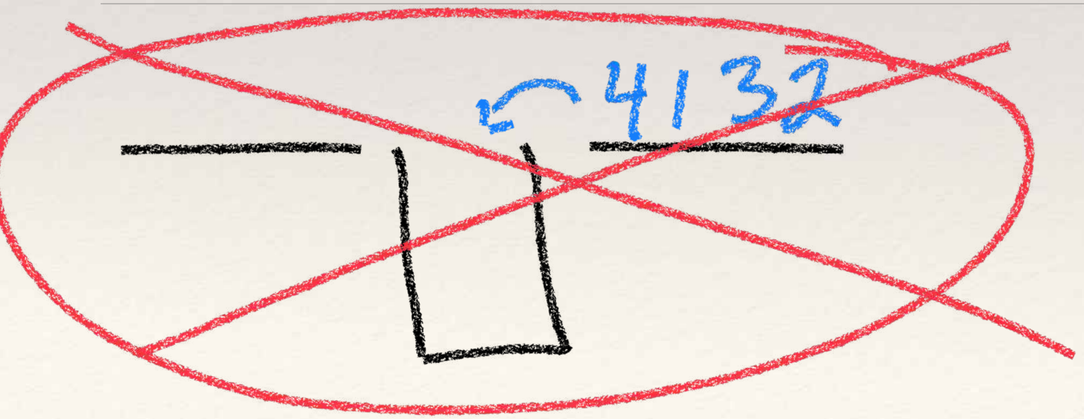

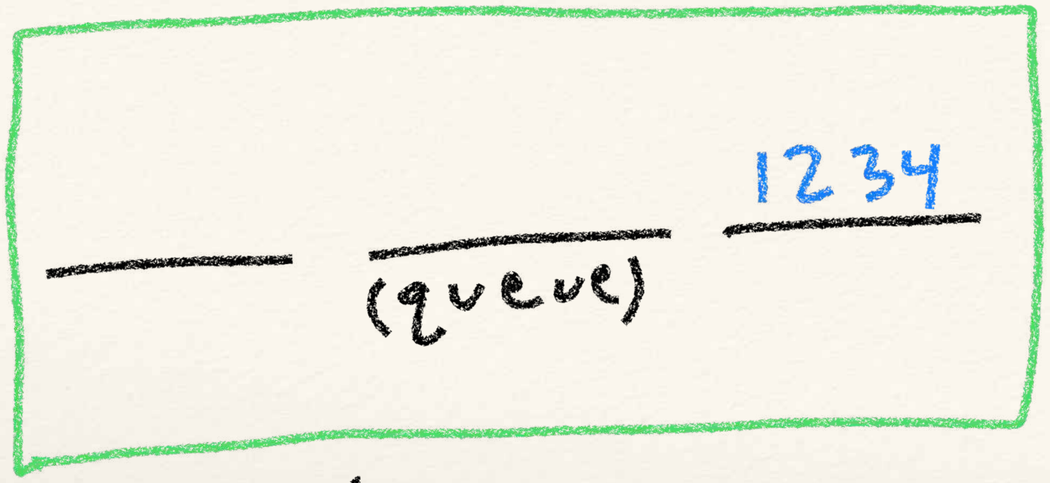

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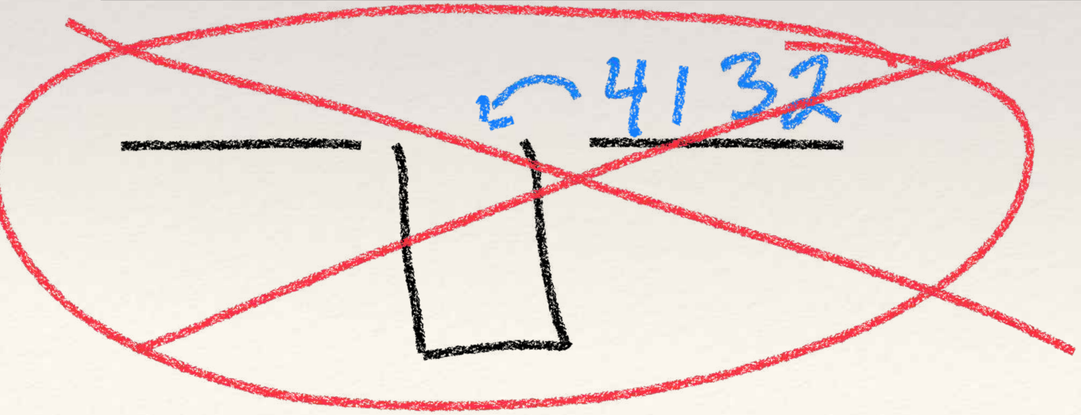

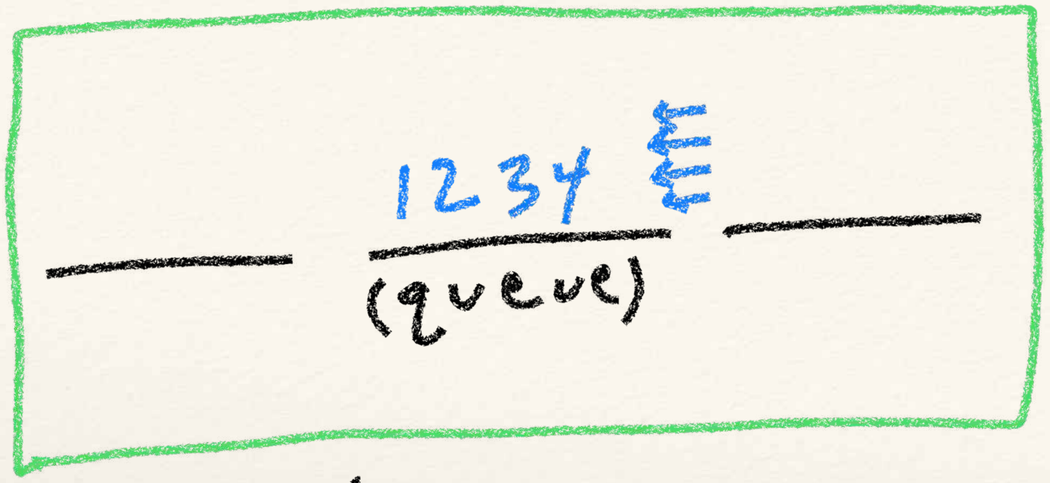

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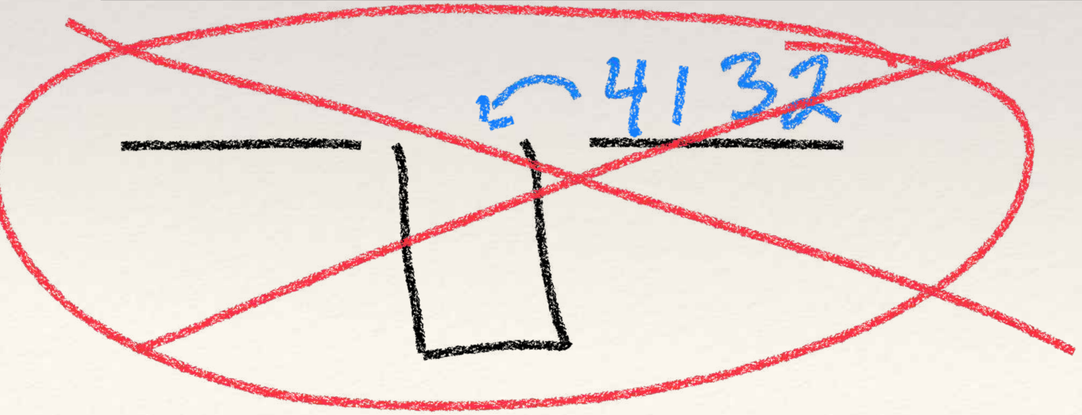

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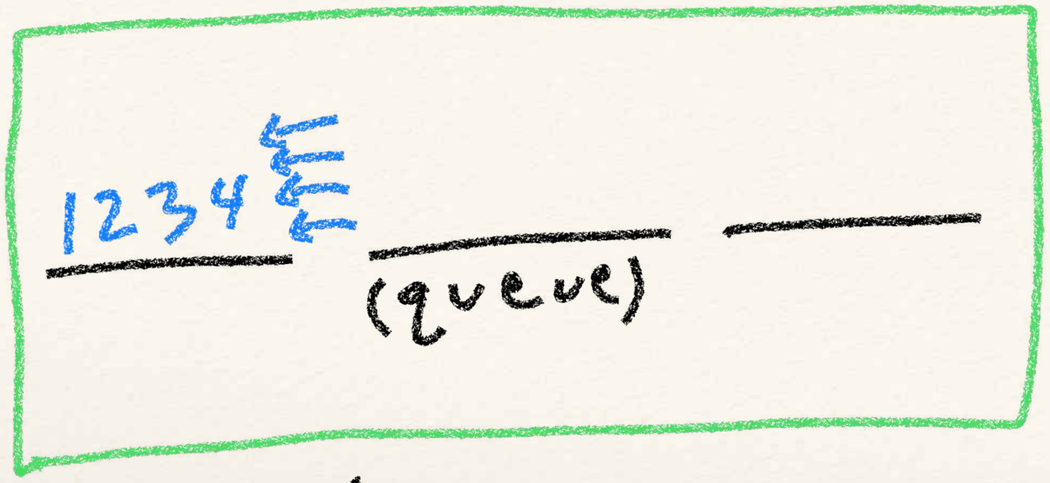

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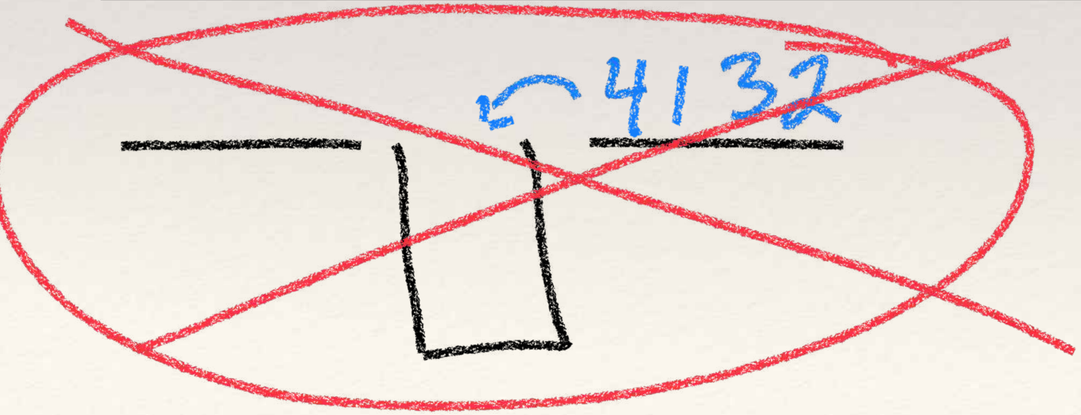

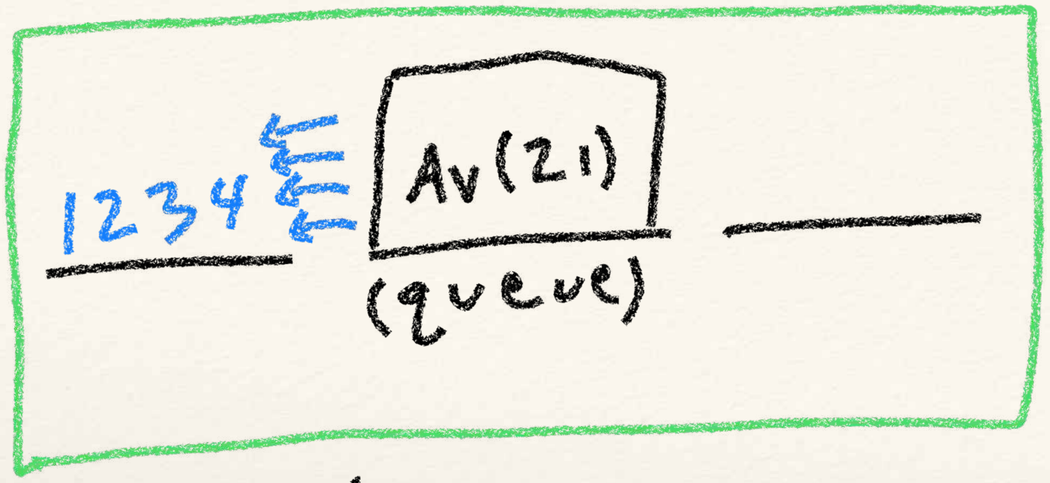

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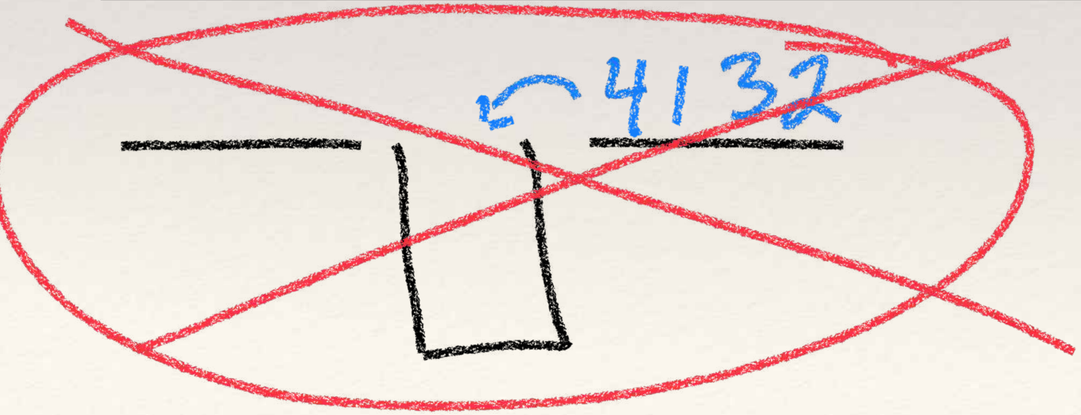
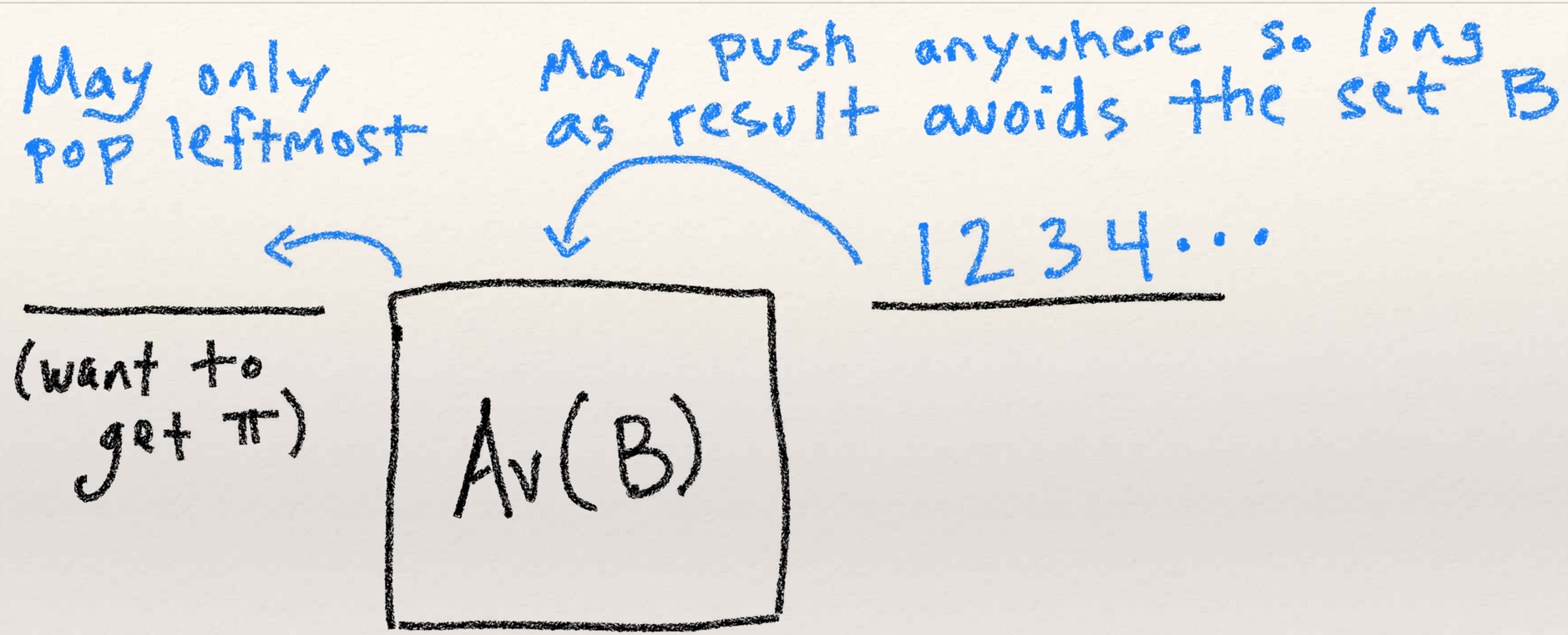


The (initial) restricted container machine



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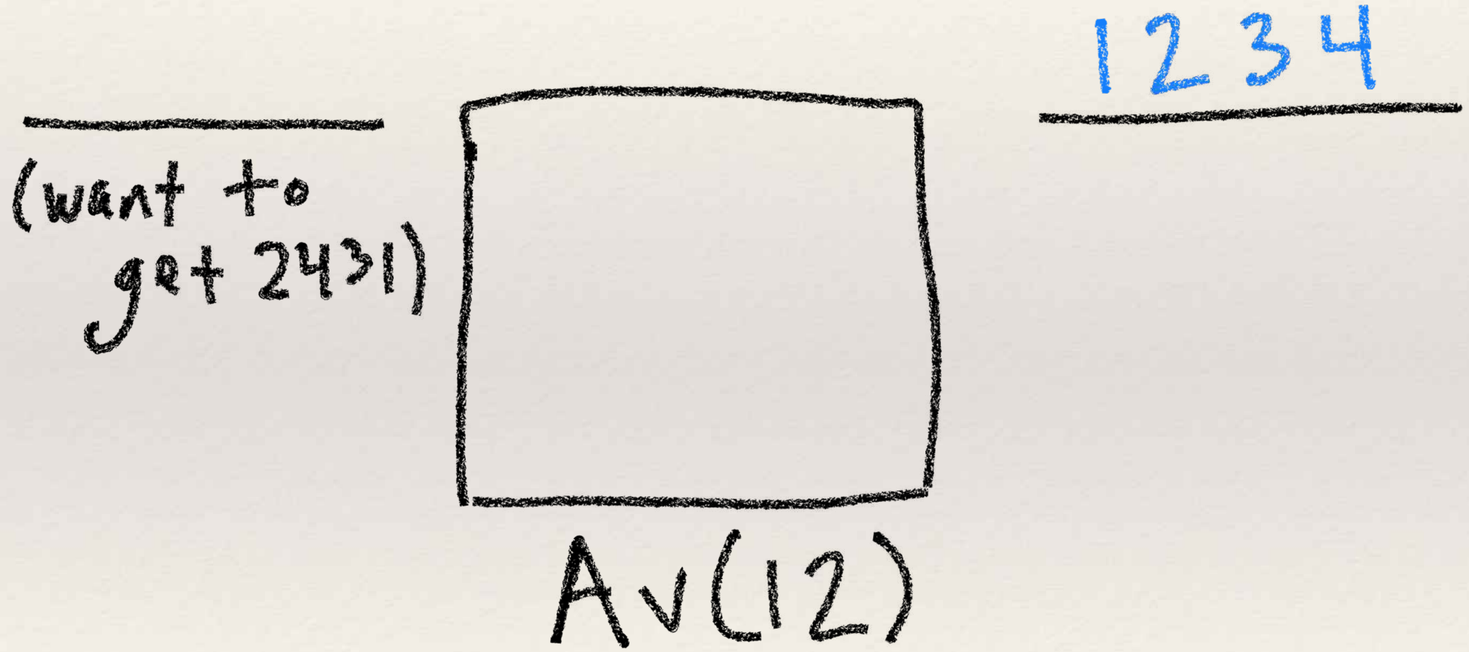

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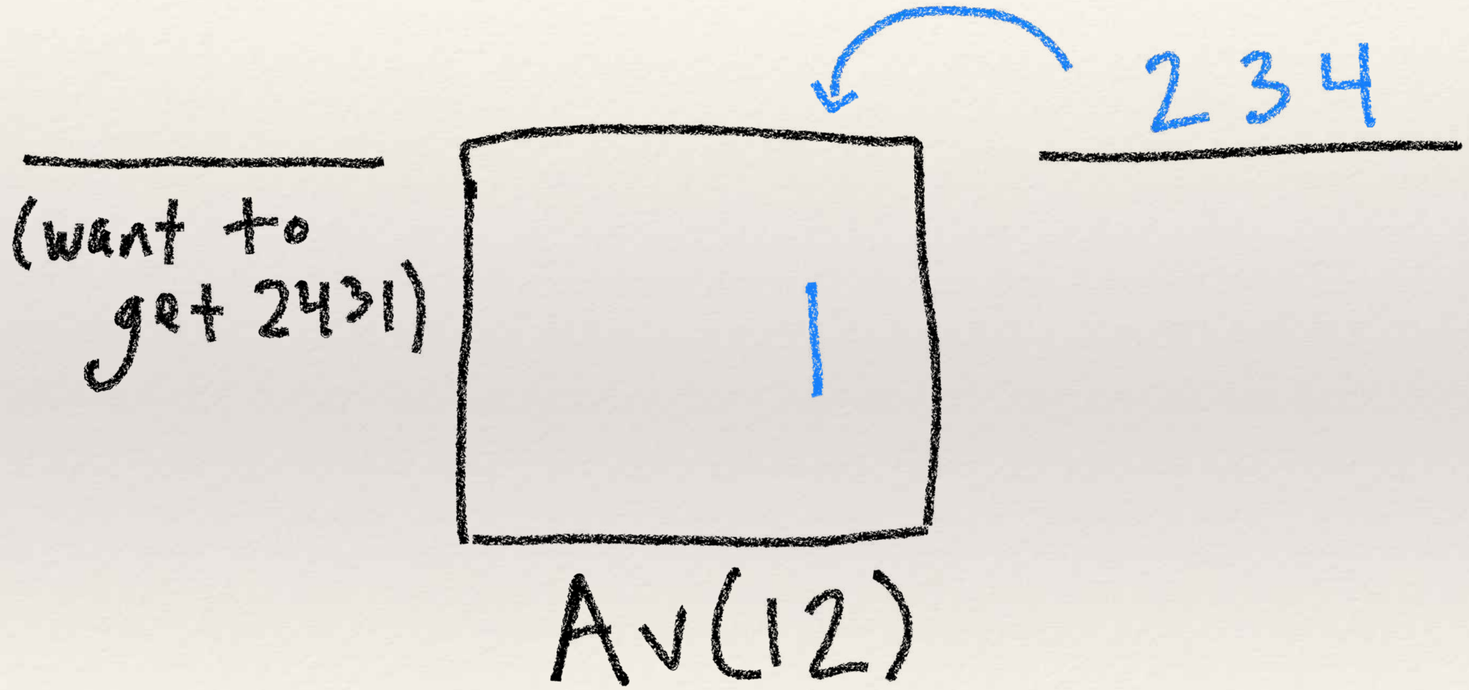

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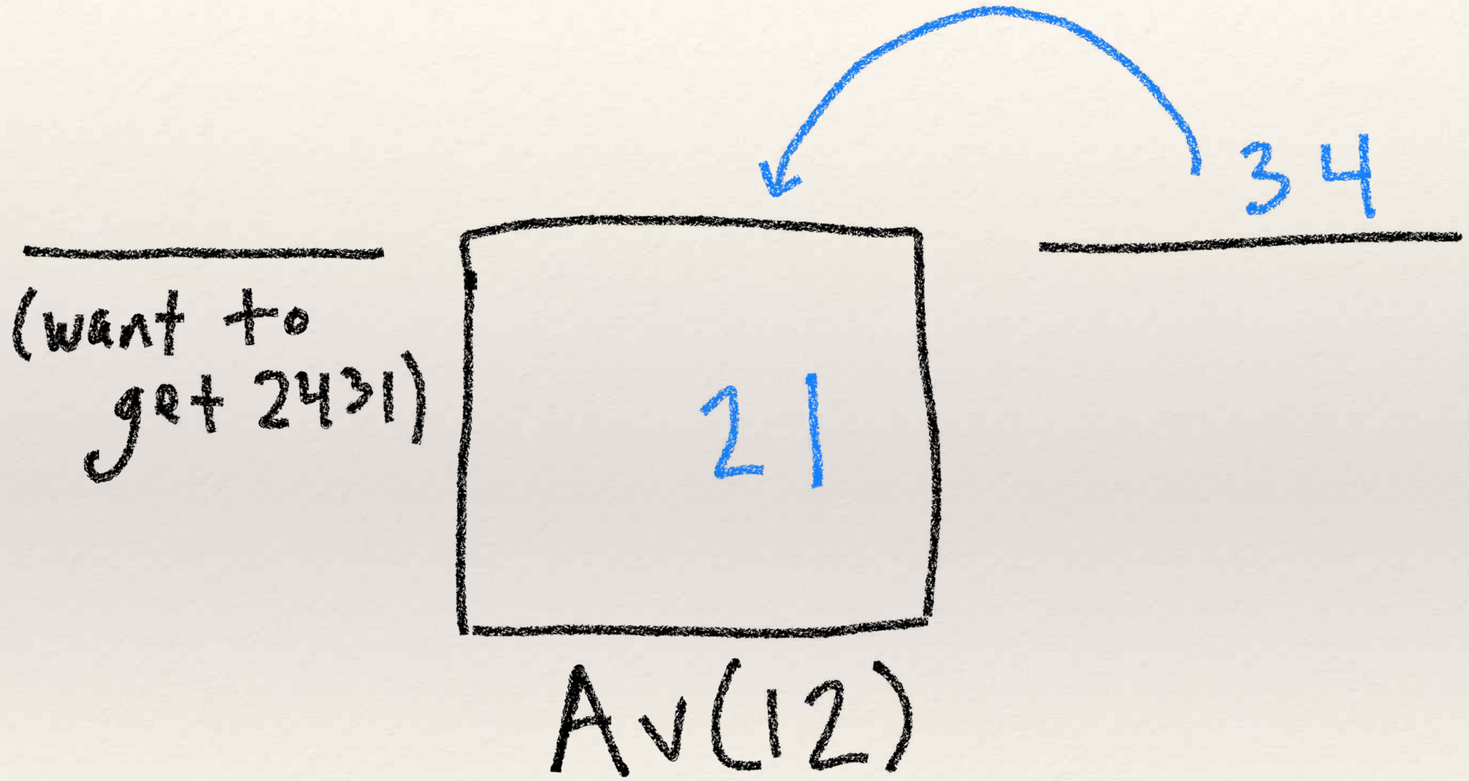

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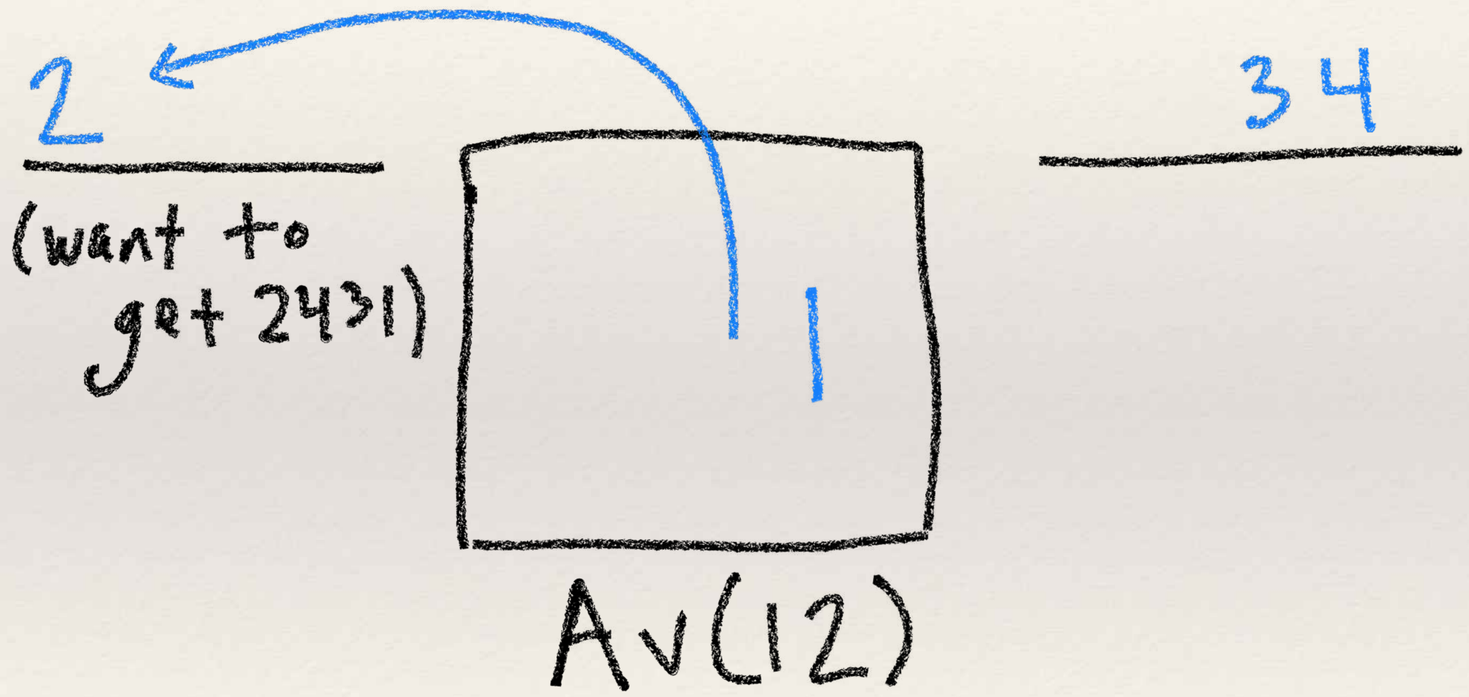

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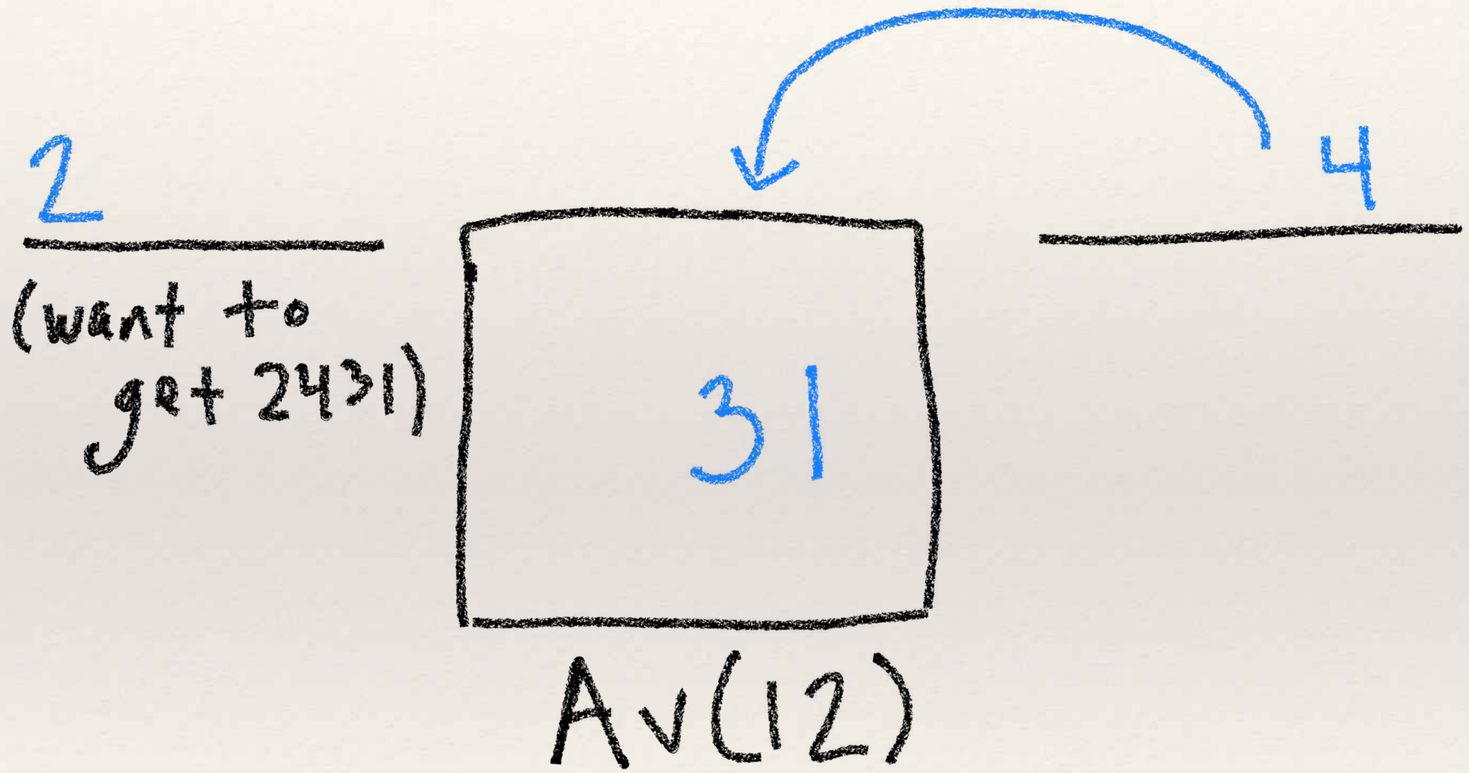

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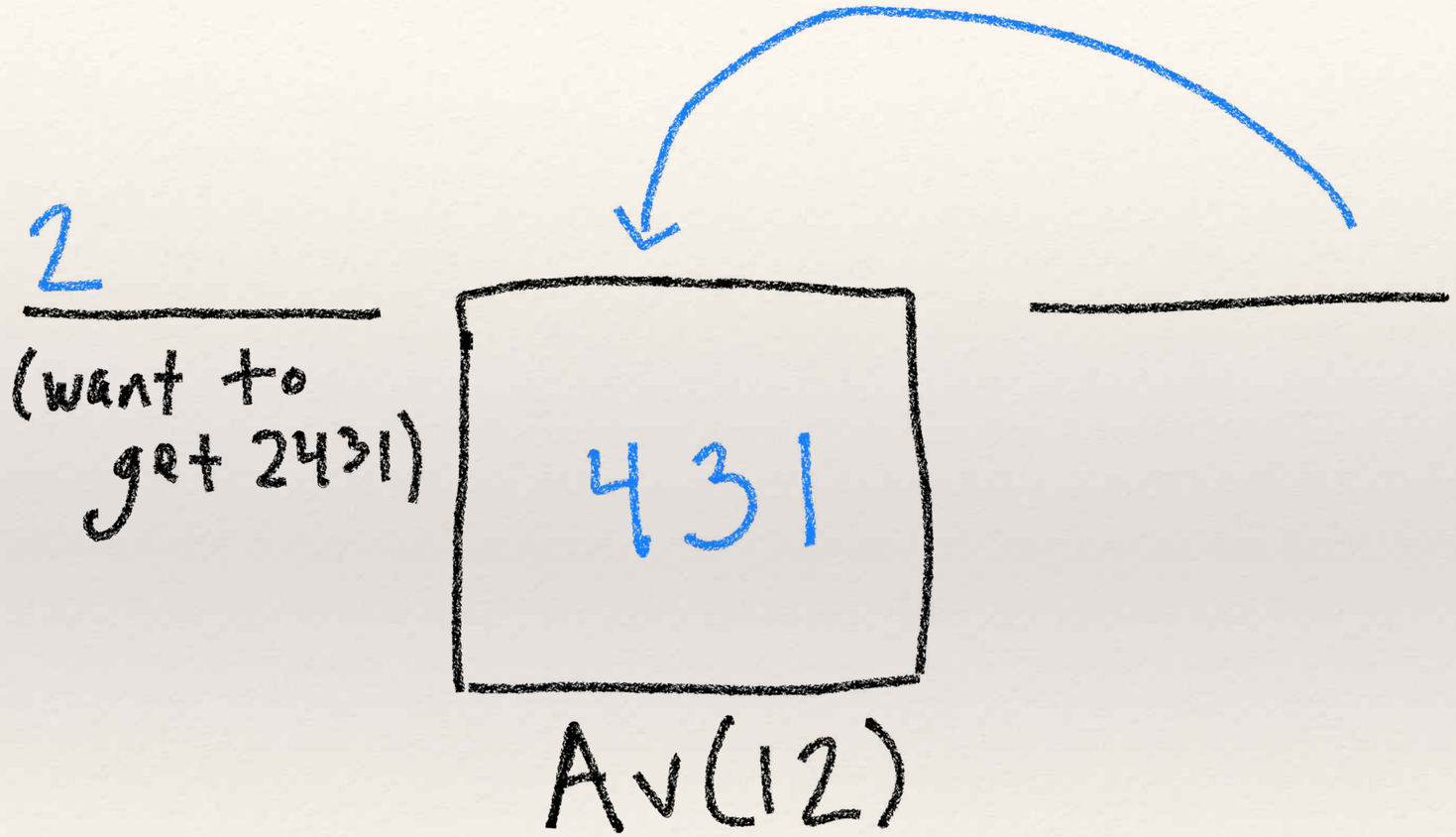

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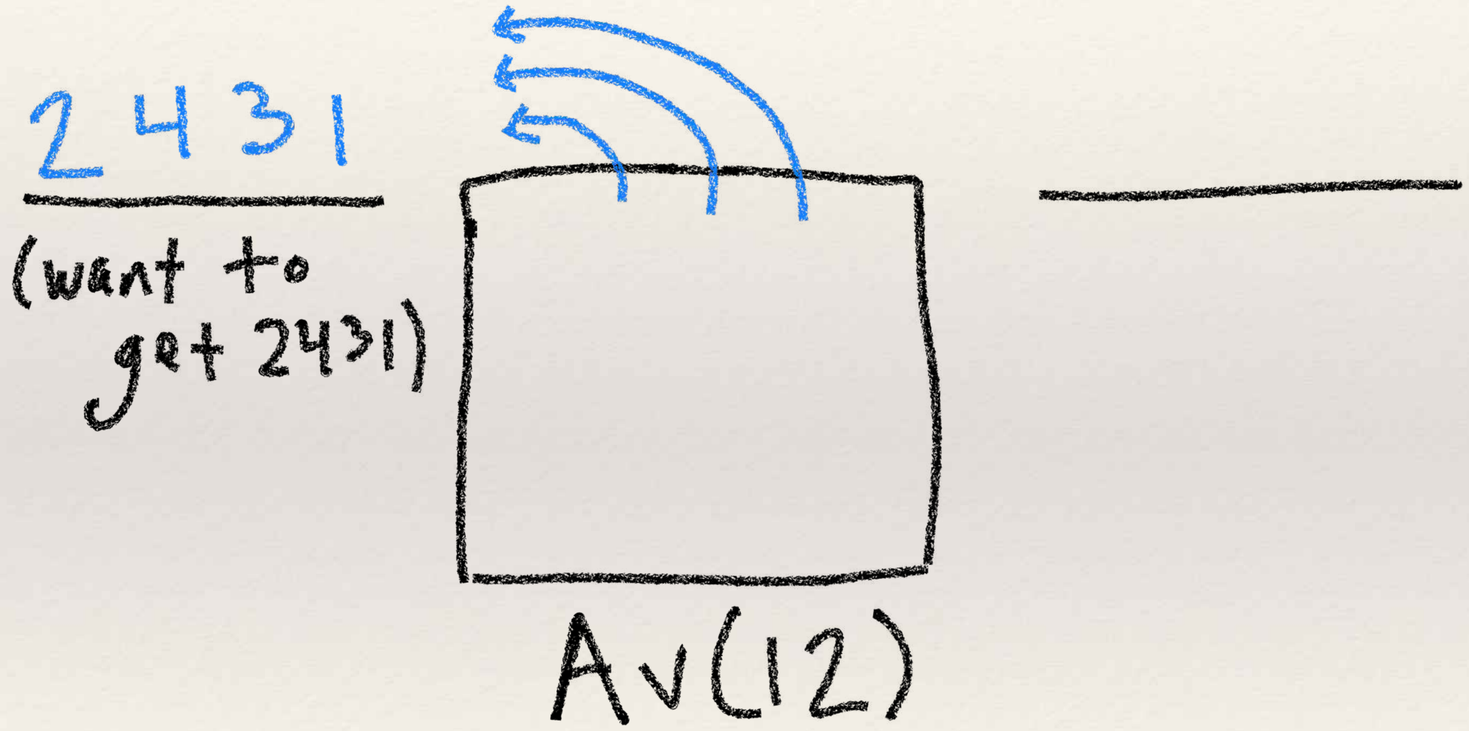

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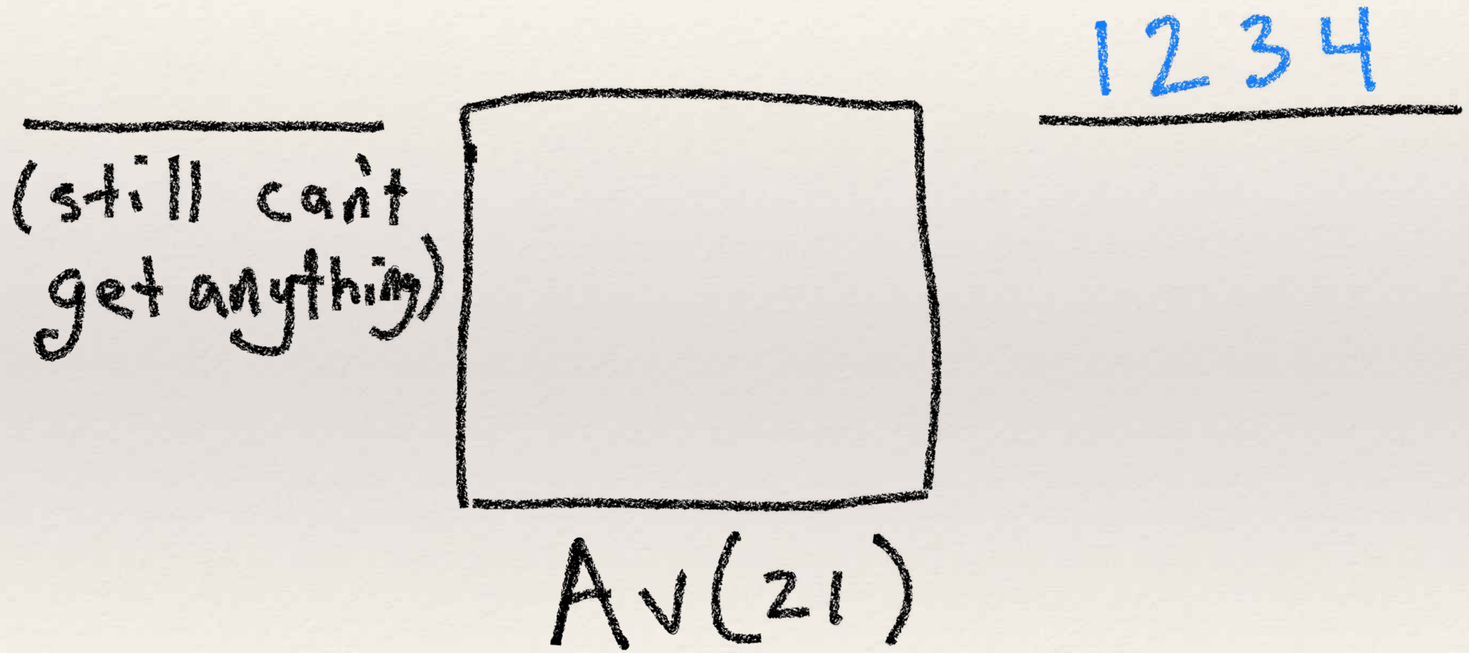

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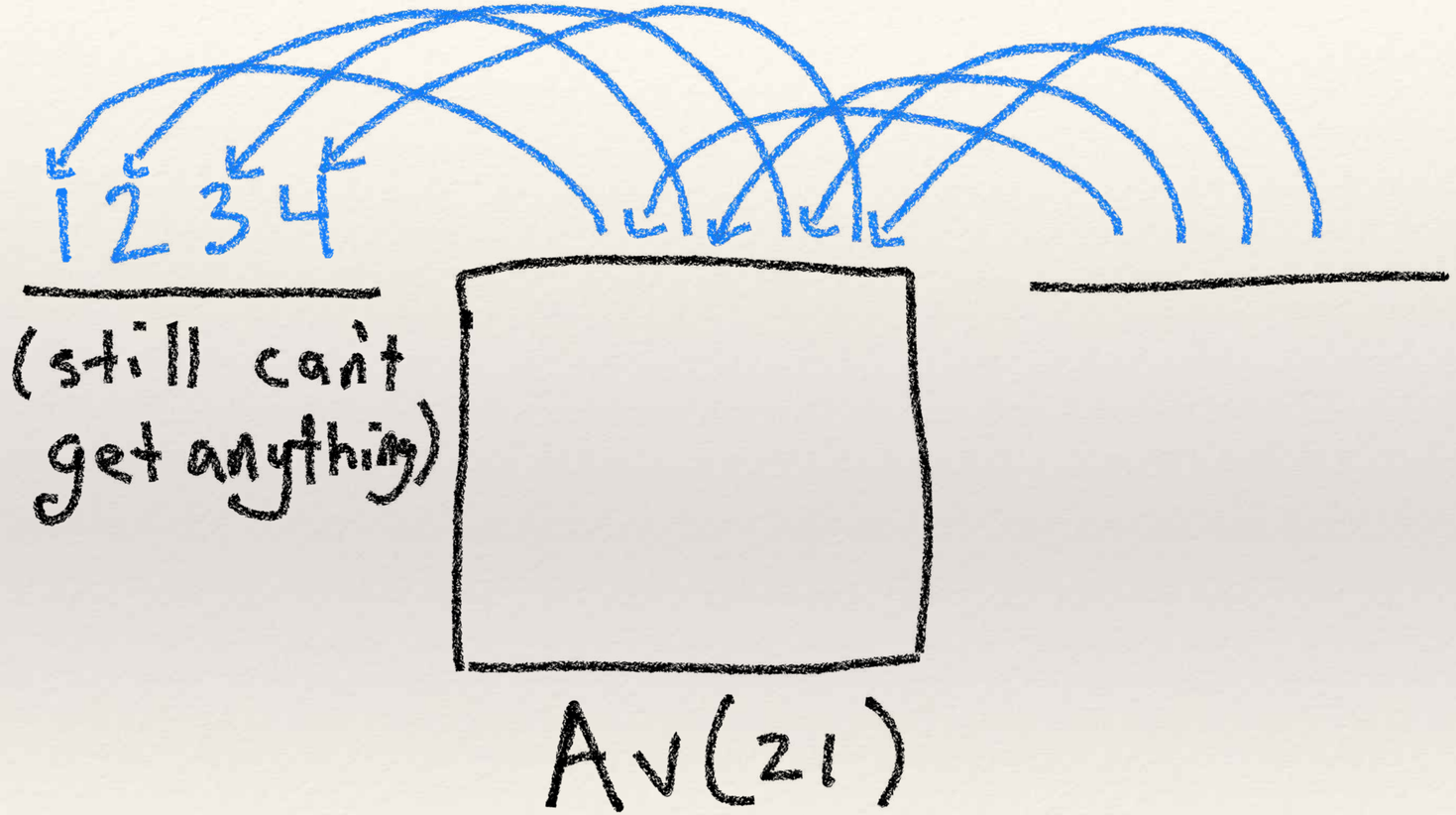

Basis theorem (initial version)

Theorem. If β does not begin with its largest entry, then the $Av(\beta)$-machine generates $Av(1 \ominus \beta)$.

prepend a new maximum
ex: 12 → 312

Basis theorem (initial version)

Theorem. If β does not begin with its largest entry, then the $Av(\beta)$-machine generates $Av(1 \ominus \beta)$.

If β does begin with its largest entry, then the $Av(\beta)$ machine just generates $Av(\beta)$.

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Basis theorem (initial version)

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If β does begin with its largest entry, then the $Av(\beta)$ machine just generates $Av(\beta)$. ☹

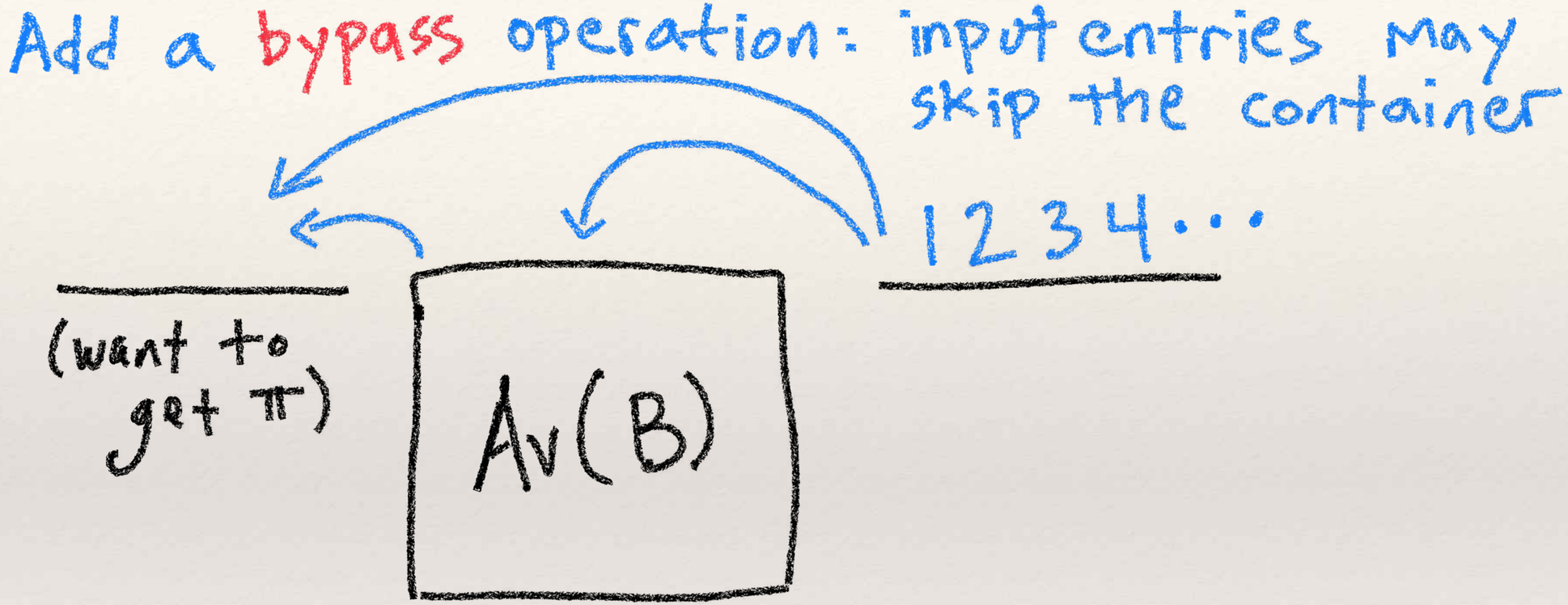
prepend a new maximum
ex: 12 → 312

Examples:

$Av(12)$-machine generates $Av(312)$ — stack

$Av(21)$-machine generates $Av(21)$ — queue

The restricted container machine



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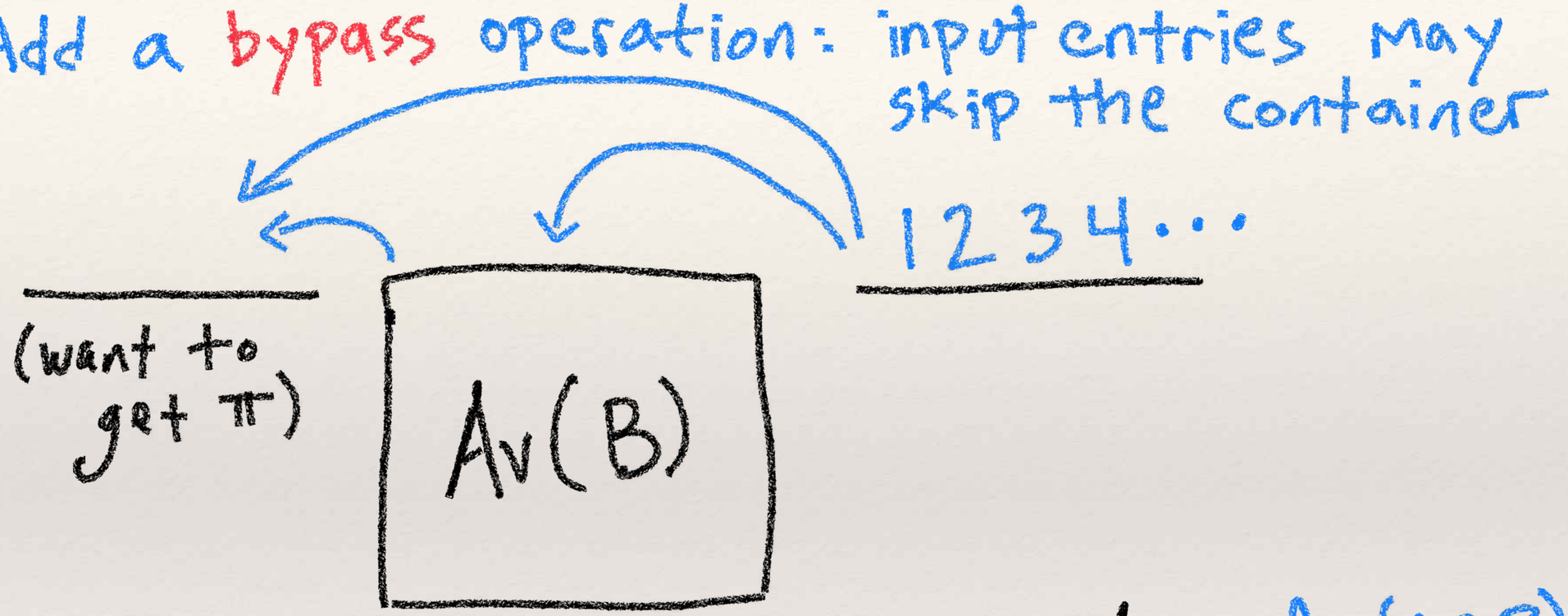

Theorem. The Av(B)-machine generates Av(1⊕B).

prepend new maximums to every member of B

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Idea. Look at left-to-right maxima of the permutation to generate.

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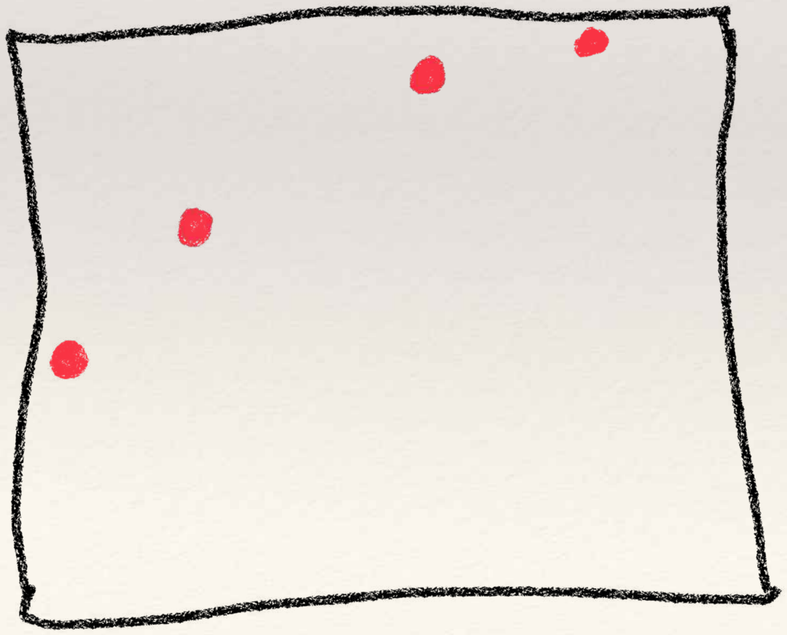

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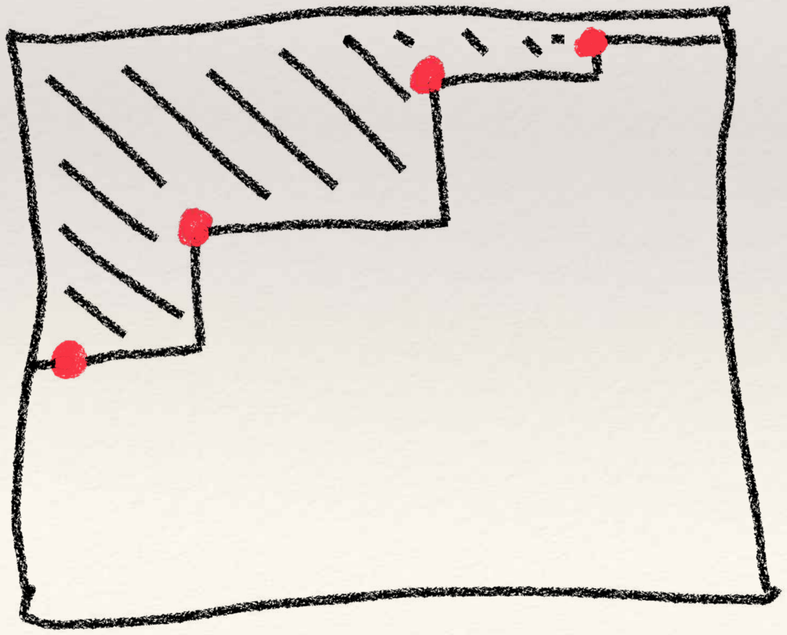

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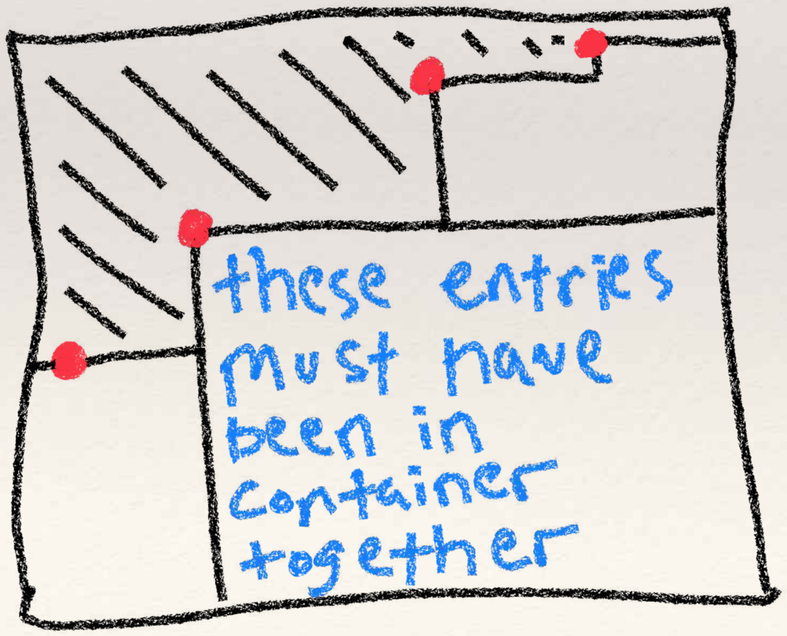
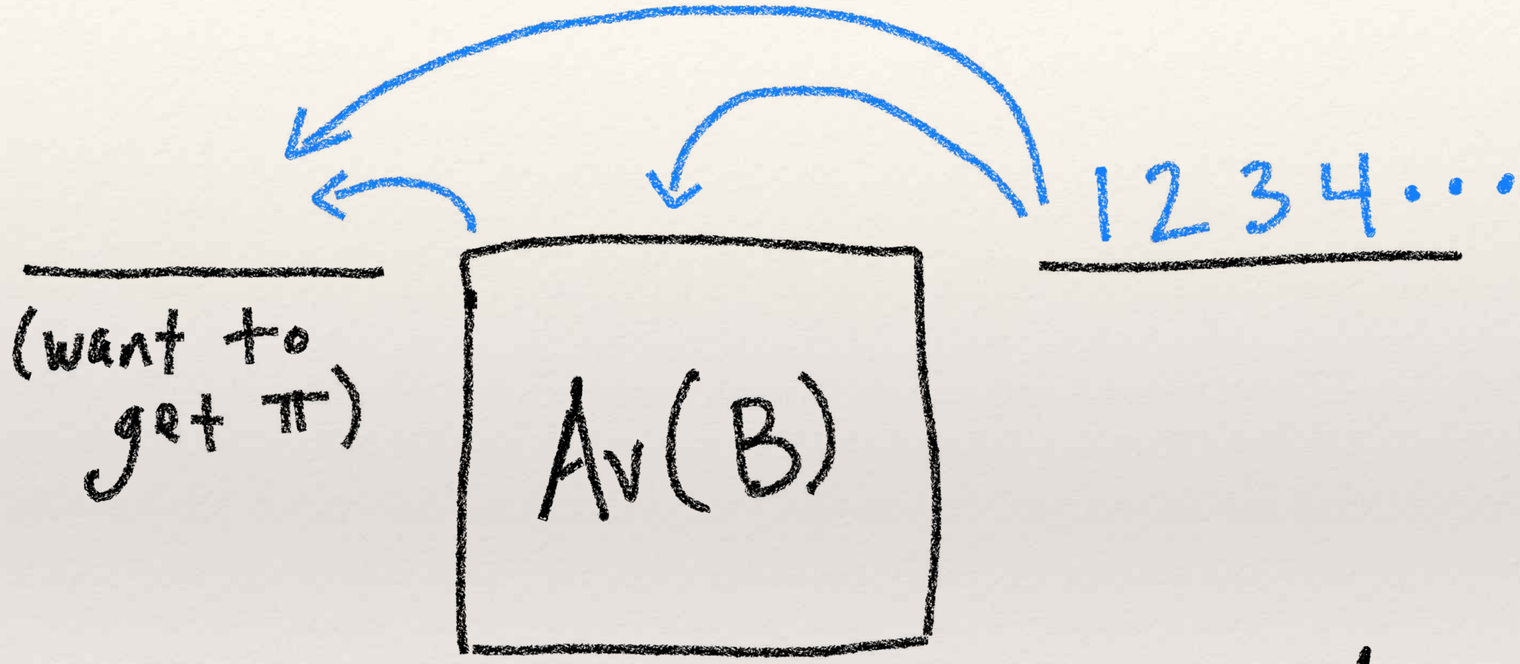


May assume all left-to-right maxima were output with bypasses.

The restricted container machine



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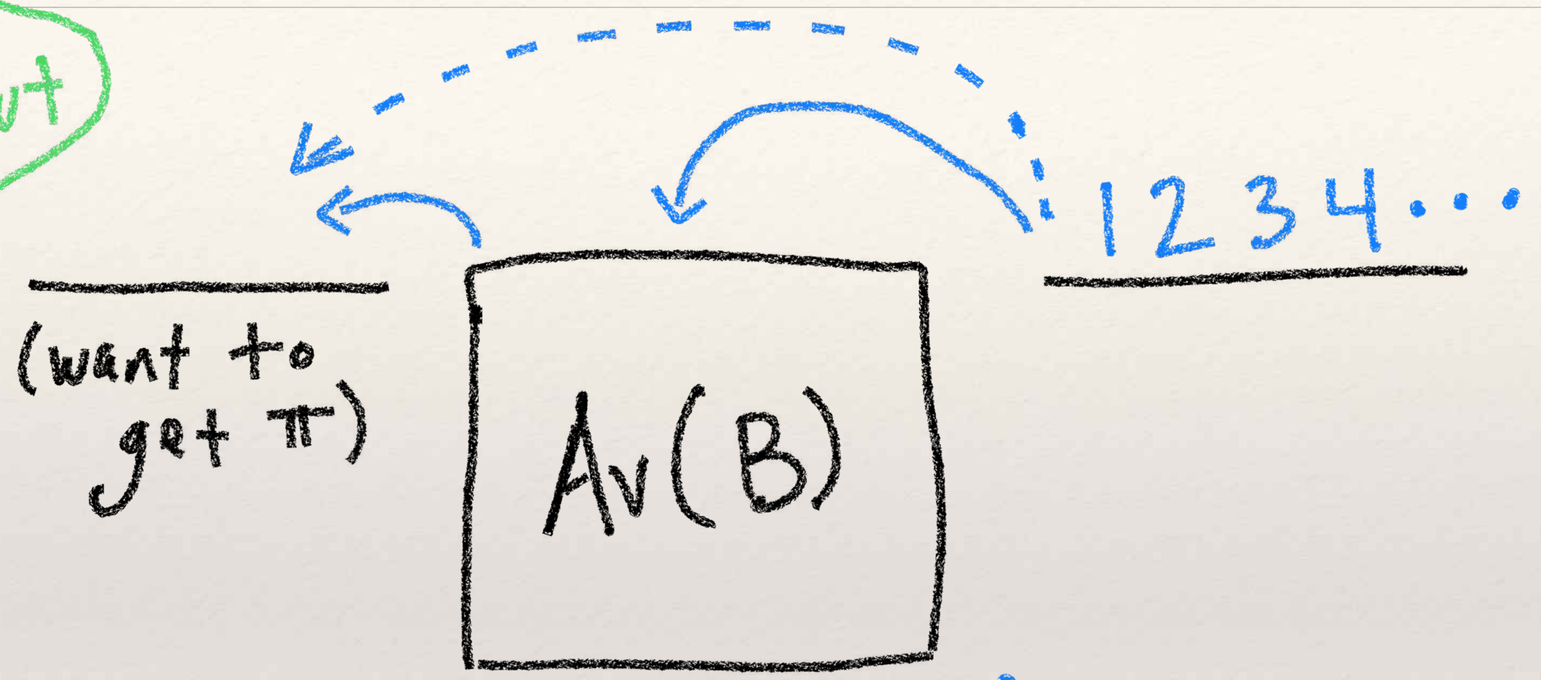
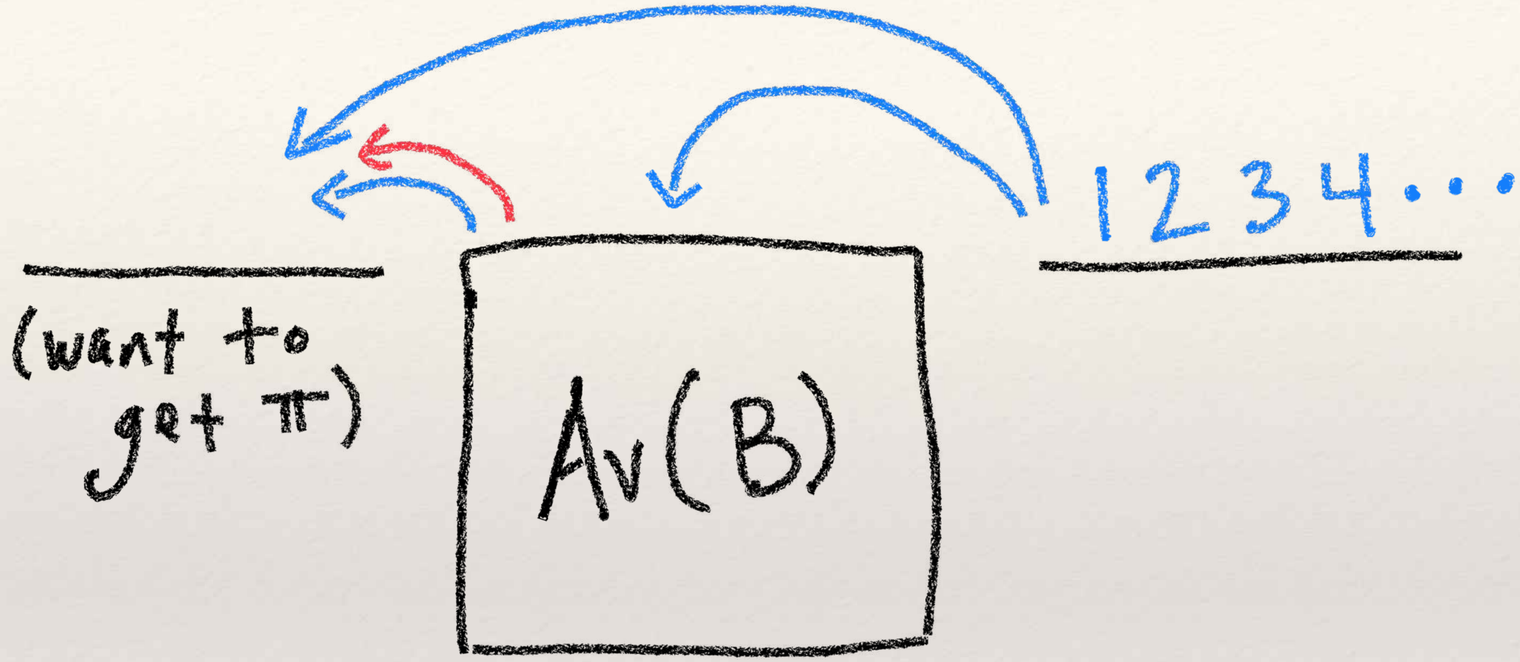
What about a "pop version" (like pop-stacks)

With/without bypasses give different answers!



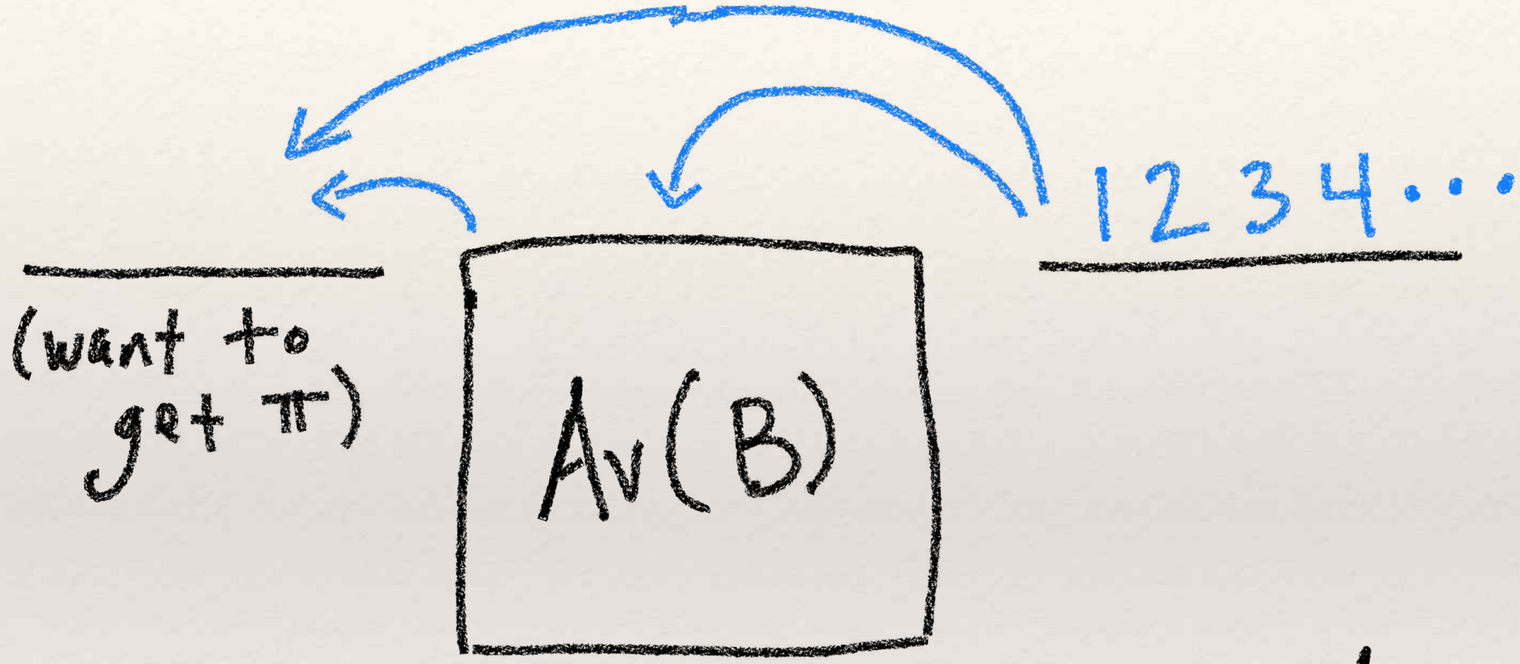
In a pop-Av(B)-machine, if you pop one entry from the container, you must empty it.

What about allowing more pops?



Could allow pops from first or second position.

The restricted container machine



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The restricted container machine

Theorem. The $\mathrm{Av}(B)$-machine generates $\mathrm{Av}(1 \oplus B)$.

Easy to see if a class can be generated!

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Bóna (PP 2007): Is there a machine that sorts $\mathrm{Av}(1342)$?

The restricted container machine

Theorem. The Av(B)-machine generates Av(1⊕B).

Easy to see if a class can be generated!

Bóna (PP 2007): Is there a machine that sorts Av(1342)?

Answer: Av(1342) ~ Av(4213), and Av(4213) is generated by Av(213)-machine.

Question: Does this explain bijection to two increasing stacks in series? (Atkinson, Murphy, Ruškuc 2002)

The restricted container machine

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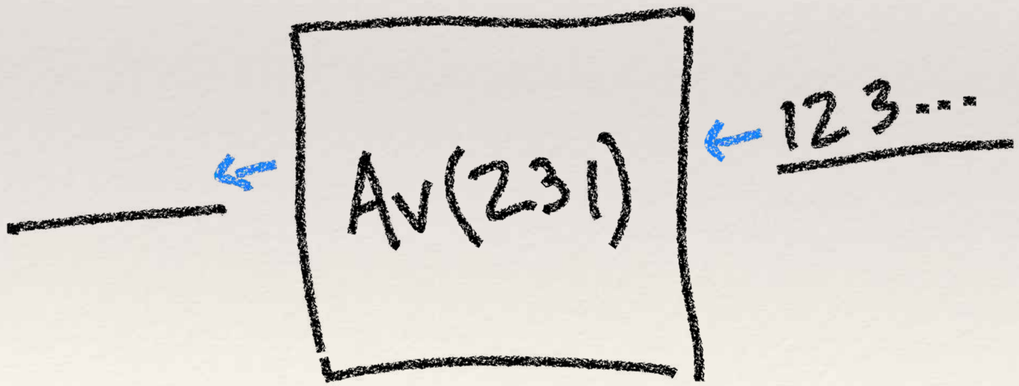
What about Av(4231)?



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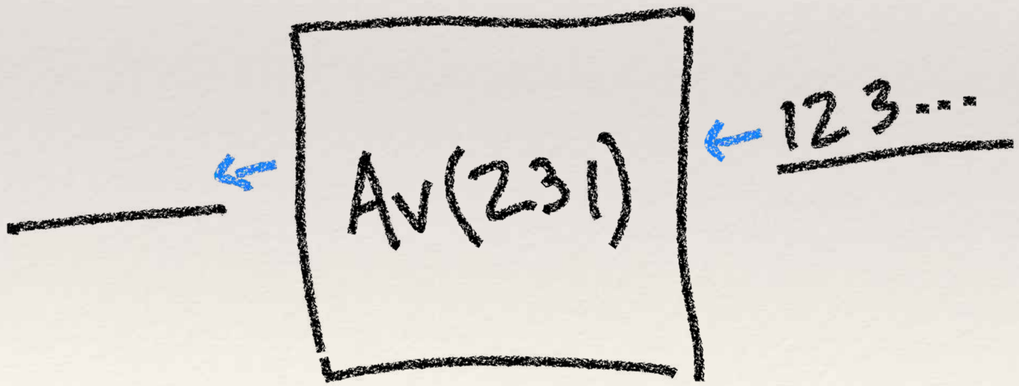

generates Av(4231)

To compute $|Av_n(4231)|$ this way takes $\approx C_n \approx 4^n$ space and time.

Conway, Guttmann, Zinn-Justin (2018) do this in about 2^n space & time.

There must be a translation of their tricks.

The restricted container machine

Theorem. The Av(B)-machine generates Av(1⊕B).

Easy to see if a class can be generated!

If the container is simple enough...

⇛ **Theorem.** generated class has rational generating function

The restricted container machine

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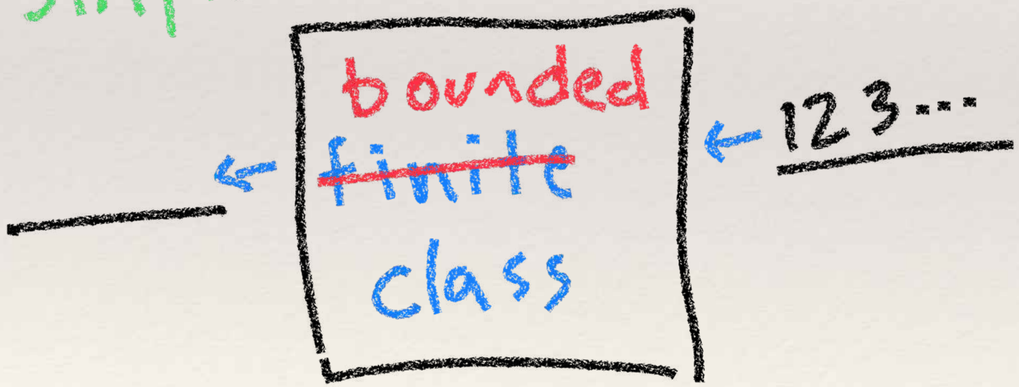


(bounded: enumeration bounded by constant)

⇒

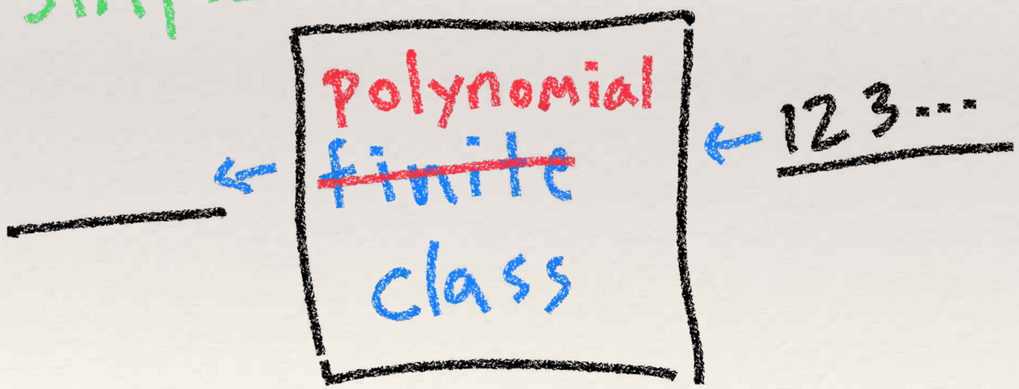
Theorem. generated class has ~~rational~~ algebraic generating function

The restricted container machine

Theorem. The Av(B)-machine generates Av(10B).

Easy to see if a class can be generated!

If the container is simple enough...



(polynomial: enumeration bounded by polynomial)

⇒

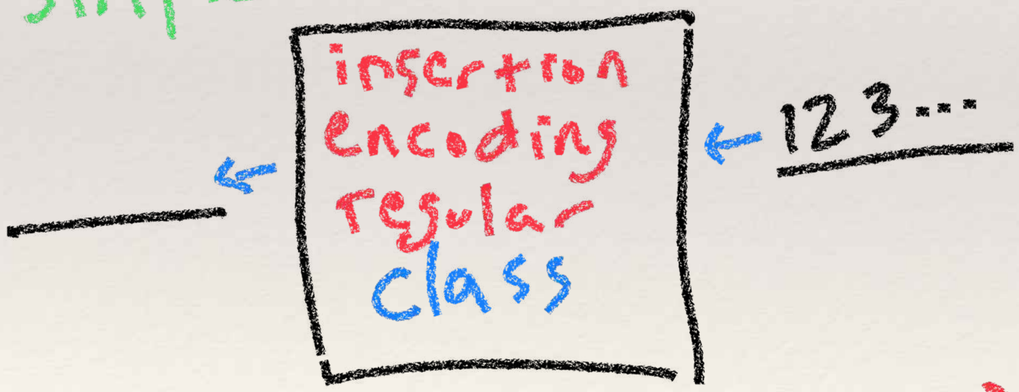
Theorem. ~~generated~~ class has ~~rational~~ ~~generating~~ ~~function~~ poly time enumeration

The restricted container machine

Theorem. The $\mathrm{Av}(B)$-machine generates $\mathrm{Av}(1 \oplus B)$.

Easy to see if a class can be generated!

If the container is simple enough...



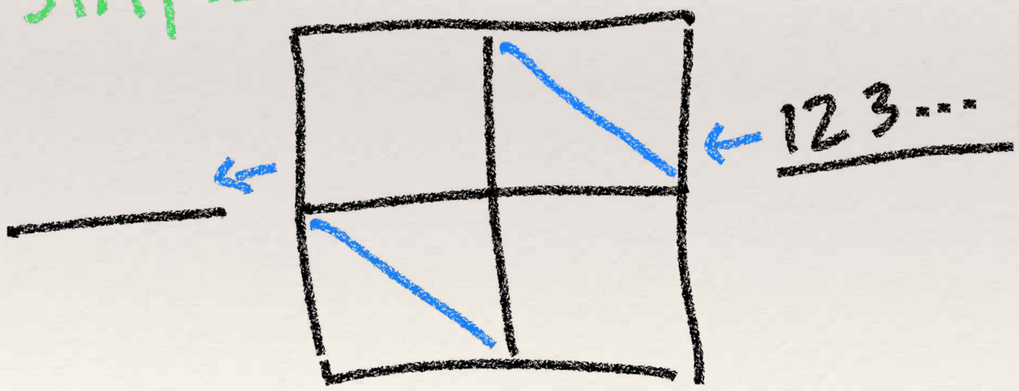
(or anything else nice)

The restricted container machine

Theorem. The Av(B)-machine generates Av(1⊕B).

Easy to see if a class can be generated!

If the container is simple enough...



Av(123, 231, 312)

⇒

5,000 terms for Av(4123, 4231, 4312)

these fit no algebraic differential equation